Quiz 9

February 22, 2017

Show all work and circle your final answer.

1. Use that $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$ to evaluate $\int \frac{2\sin\theta\cos\theta}{\sqrt{5-\sin\theta}}\,d\theta.$

$u = \sin\theta$
$du = \cos\theta\, d\theta$

$$= 2\int \frac{u}{\sqrt{5-u}}\,du \underset{\substack{a=5\\ b=-1}}{=} 2\left[\frac{2}{3(-1)^2}(-u-2(5))\sqrt{5-u} + C\right]$$

$$= \boxed{\frac{4}{3}(-\sin\theta-10)\sqrt{5-\sin\theta}+C}$$

2. Given the following table, write a formula to approximate $\int_0^8 f(x)\,dx$ using 4 subintervals and

(a) Simpson's Method

(b) Midpoint Rule

x	0	1	2	3	4	5	6	7	8
$f(x)$	π	1	-3	2	7	e	-4	-2	3

For each method, $\Delta x = \frac{8-0}{4} = 2$. I'll use $I = \int_0^8 f(x)dx$.

(a) $I \approx \frac{\Delta x}{3}\left[f(0)+4f(2)+2f(4)+4f(6)+f(8)\right] = \boxed{\frac{2}{3}\left[\pi+4(-3)+2(7)+4(-4)+3\right]}$

(b) $I \approx \Delta x\left[f(1)+f(3)+f(5)+f(7)\right] = \boxed{2\left[1+2+e-2\right]}$

3. Evaluate $\int_0^3 \frac{1}{x-1}\,dx.$ Notice $y=\frac{1}{x-1}$ has a vertical asymp. at $x=1$.

$$= \lim_{t\to1^-}\int_0^t \frac{1}{x-1}dx + \lim_{s\to1^+}\int_s^3 \frac{1}{x-1}dx$$

$$= \lim_{t\to1^-}\left[\ln|x-1|\right]_0^t + \lim_{s\to1^+}\left[\ln|x-1|\right]_s^3$$

$$= \lim_{t\to1^-} \ln|t-1| - \ln 1 + \lim_{s\to1^+} \ln 2 - \ln|s-1|$$

DNE since $\lim_{u\to0^+}\ln u$ DNE

$\boxed{\text{The integral diverges}}$